On the nonlinear stage of Modulation Instability

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Abstract:

It is known since 1971 that the Nonlinear Schrödinger (NLSE) is a system completely integrable by the Inverse Scattering Method (ISTM). The NLSE has a simple solution, the monochromatic wave with frequency depended on amplitude - the condensate. The condensate is unstable with respect to modulation instability. There are important question: what is a nonlinear stage of modulation instability? In spatial dimension $D = 2, 3$, the answer is known - modulation instability leads to formation of finite time singularities - collapses. In dimension $D = 1$ collapses are forbidden. However in this case development of modulation instability leads to formation of "extreme" (rogue, freak) waves where energy density exceeds the mean level by order of magnitude.

In this join work with professor V. E. Zakharov [1] we study solitonic solutions of the focusing NLSE in the presence of the condensate by using the dressing method. We find a general N-solitonic solution and separate a special designated class of "regular solitonic solutions" that do not disturb phases of the condensate at infinity by coordinate. All regular solitonic solutions can be treated as localized perturbations of the condensate. If we assume that the modulation instability develops from localized perturbation, only regular solution can be used as model for its nonlinear behavior.

The central result of our work is following. We find a broad class of "superregular solitonic solutions" which are small perturbations at certain a moment of time. Then they develop into N pairs of different solitons (we call them "quasi-Akhmediev" breathers). These solutions form an infinite-dimensional linear functional space. This describes the nonlinear stage of the modulation instability of the condensate and can be treated as a sort of "integrable turbulence" where local concentration of energy easily exceeds in order of magnitude the energy density in the condensate. Self-consistent analytic theory of this turbulence will be a truly reliable theory of freak waves.

References:

1. V.E. Zakharov and A.A. Gelash, On the nonlinear stage of Modulation Instability, arXiv:1212.1393 (2012).